

Problem Set 0

Discrete Structures

31st day of August of the year of our Lord 2026

Justify each of your answers with an argument. An unjustified answer is worse than incorrect.

1. Determine the truth value, if any, of each of the following sentences.

- a. "Every integer is either even or odd."¹
- b. "Every non-empty set of integers has a smallest element."²
- c. "Every non-empty set of natural numbers has a smallest element."³
- d. "Every natural number is boring."⁴
- e. "No natural number is boring."

¹ Informally, an *integer* is either a positive whole number 1, 2, 3, ... or a negative whole number -1, -2, -3, ... or zero 0.

² Informally, a *set* is a "collection containing some number of objects."

³ Informally, a *natural number* is a non-negative whole number 0, 1, 2, ...

⁴ For some reasonable definition of "boring."

2. Determine the truth value, if any, of each of the following sentences.⁵

- a. "This sentence is *true*."⁶
- b. "This sentence is *false*."
- c. "The set of all sets that contain themselves contains itself."
- d. "The set of all sets that do not contain themselves does not contain itself."
- e. "If this sentence is *false*, then 7 is a prime number."
- f. "If this sentence is *true*, then 7 is not a prime number."

⁵ You may assume that 7 is prime.

⁶ You may find it helpful to give a sentence like this a name. For example, define $S := \text{"}S \text{ is true.}"$ The sentence S is equivalent to " $\text{"}S \text{ is true,}"$ " so reason about S instead.

3. Fix a universe of discourse consisting of all of the characters in Shakespeare's play *Macbeth*, and consider the following predicate definitions.

$\kappa(x) := \text{"}x \text{ is a King.}"$

$\beta(x) := \text{"}x \text{ is a man woman-borne.}"$

$\vartheta(x) := \text{"}x \text{ is a Thane.}"$

$\mu(x, y) := \text{"}x \text{ murders } y."$

$v(x) := \text{"}x \text{ is a witch.}"$

Translate each of the following English sentences into symbolic expressions. Although there are many *logically equivalent* ways to symbolize each of these sentences, you should try to remain faithful to the original sentence structure.

- a. "Macbeth is a King, and Banquo is a Thane."
- b. "Every King is murdered by someone."
- c. "Someone murders every King."
- d. "No witches murder anyone."
- e. "No Kings murder any Thanes."
- f. "Macbeth is not murdered by any man woman-borne."
- g. "Every King is a Thane."
- h. "Banquo is not a King; yet, he was murdered."