Problem Set 1

Discrete Structures

Due on the 1ˢᵗ day of February of the year of our Lord 2026 at 11:59 pm

1. Prove each of the following statements *without truth tables.*[1]

   a. Prove $p \to p$ is a tautology for any proposition p.

   b. Prove $p \to q \equiv \neg q \to \neg p$ for all propositions p, q.

   c. Prove $\neg(p \to q) \equiv p \wedge \neg q$ for all propositions p, q.

   d. Prove $\big(p \wedge (p \to q)\big) \to q$ is a tautology for all propositions p, q.

   e. Prove $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$ for all propositions p, q, r.

2. Prove each of the following statements for all propositions φ, ψ, ξ.

   a. $(\varphi \to \psi), (\psi \to \xi) \vdash \varphi \to \xi$ *Hypothetical Syllogism*

   b. $\vdash \varphi \to \varphi$

   c. $\vdash \top$

   d. $\varphi, \psi \vdash \varphi \wedge \psi$ *Conjunction Introduction*

   e. $\varphi \wedge \psi \vdash \varphi$ *Conjunction Elimination*

[1] In addition to the axioms of propositional logic and the uniqueness of complements, you may assume the following theorems:
1. $\top \equiv \neg\bot$ and $\bot \equiv \neg\top$
2. Double Negation
3. Idempotence
4. Domination
5. De Morgan's Laws