Problem Set 0

Discrete Structures

Due on the 25th day of January of the year of our Lord 2026 at 11:59 pm

Justify each of your answers with an argument. An answer provided with bad or no justification is worse than a wrong answer.

1. Determine the truth value, if any, of each of the following sentences.
   a. "Every integer is either even or odd."[1]
   b. "Every number is either even or odd."
   c. "Every non-empty set of integers has a smallest element."[2]
   d. "Every non-empty set of natural numbers has a smallest element."[3]
   e. "Every natural number is boring."[4]
   f. "No natural number is boring."
2. Determine the truth value, if any, of each of the following sentences.[5]
   a. "This sentence is *true*."[6]
   b. "This sentence is *false*."
   c. "The set of all sets that contain themselves contains itself."
   d. "The set of all sets that do not contain themselves does not contain itself."
   e. "If this sentence is *false*, then 7 is a prime number."
   f. "If this sentence is *true*, then 7 is not a prime number."
3. Consider the infinite sequence of sentences indexed by natural numbers

   $$S_0, S_1, S_2, ..., S_i, ...$$

   where each sentence asserts that all of the subsequent sentences are *false*.[7]

   $$S_i := \text{"}S_j \text{ is } \textit{false} \text{ for all } j > i\text{"}$$

   For example, $S_3 :=$ "S_j is *false* for all $j > 3$," which means S_3 is equivalent to the assertion that "the sentences $S_4, S_5, S_6, ...$ are all *false*."

   What are the truth values, if any, of the sentences in this sequence?[8][9]

[1] Informally, an *integer* is either a positive whole number $1, 2, 3, ...$ or a negative whole number $-1, -2, -3, ...$ or zero 0.

[2] Informally, a *set* is a "collection of objects."

[3] Informally, a *natural number* is a non-negative whole number $0, 1, 2, ...$

[4] For some reasonable definition of "boring."

[5] You may that assume 7 is actually a prime number.

[6] You may find it helpful to give a sentence like this a name. For example, define $S :=$ "S is *true*." The sentence S is equivalent to "This sentence is *true*.", so reason about S instead.

[7] The variable i in the definition of S_i ranges over all of the natural numbers $0, 1, 2, ...$

[8] Your argument should describe the truth values, if any, of *every* sentence in this sequence.

[9] HINT: you should first determine what the truth value of S_0 is; once you have worked this out, then you can think about generalizing your argument to an arbitrary sentence S_i in the sequence.