NAME: NETID:

Practice Midterm 1

Discrete Structures

1st day of April of the year of our Lord 2026

1 *Answer each of the following questions by marking True or False but not both.*

1. $(\forall n \in \mathbb{N})(\varphi(n)) \Leftrightarrow (\forall k \in \mathbb{N})\Big((\forall \ell \in \mathbb{N})\big((\ell < k) \Rightarrow \varphi(\ell)\big) \Rightarrow \varphi(k)\Big)$ for any *wff* φ.

   ○ *True*
   ○ *False*

2. Every positive natural number has a prime divisor.

   ○ *True*
   ○ *False*

3. If p is prime, then $(\forall x \in \mathbb{Z})\big(x \mid p \;\Rightarrow\; x \in \{1, p\}\big)$.

   ○ *True*
   ○ *False*

4. $(\forall \mathcal{Z} \subseteq \mathbb{Z})\big(\mathcal{Z} \neq \varnothing \;\Rightarrow\; (\exists z \in \mathcal{Z})(\forall w \in \mathbb{Z})(z \leqslant w)\big)$.

   ○ *True*
   ○ *False*

5. $\mathbb{Z} = \Big\{z \in \mathbb{Z} \;\Big|\; (\exists x, y \in \mathbb{Z})(z = 76025 \cdot x + 76 \cdot y)\Big\}$.

   ○ *True*
   ○ *False*

6. $x \cdot y = \gcd(x, y) \cdot \operatorname{lcm}(x, y)$ for all $x, y \in \mathbb{N}$.

   ○ *True*
   ○ *False*

7. $\big(\forall x, y \in \mathbb{N}_+\big)\big(x \mid y \;\Leftrightarrow\; |x| \leqslant |y|\big)$.

   ○ *True*
   ○ *False*

8. No integer has a multiplicative inverse.

   ○ *True*
   ○ *False*

9. For all $x, y, z \in \mathbb{N}$, if x and y are coprime and $x \mid y \cdot z$, then $x \mid z$.

   ○ *True*
   ○ *False*

10. If $n \in \mathbb{N}$ and $n > 0$, then $\cup n$ is the predecessor of n.

    ○ *True*
    ○ *False*

2 *You may assume that addition and multiplication are commutative and associative on* $\mathbb{N}$*, that* 0 *is the additive identity, that* 1 *is the multiplicative identity, and that* $\mathfrak{suc}(x) = x + 1$ *for all* $x \in \mathbb{N}$*. You may not assume anything discussed after these properties were mentioned.*

Arguing directly from the recursive definitions of addition, multiplication, and exponentiation on $\mathbb{N}$, prove that $(\forall x, n, m \in \mathbb{N})\big(x^n x^m = x^{n+m}\big)$.

3 *You may assume all theorems up to and including the Fundamental Theorem of Arithmetic, Euclid's Division Lemma, Bézout's Identity, and Euclid's Lemma.*

Prove that n and $n + 1$ are coprime for all $n \in \mathbb{N}$.

4 *You may assume all theorems up to and including the Fundamental Theorem of Arithmetic, Euclid's Division Lemma, Bézout's Identity, and Euclid's Lemma.*

Use induction to prove that $(\forall n \in \mathbb{N})(5 \mid 8^n - 3^n)$.