NAME: NETID:

Practice Midterm 0

Discrete Structures

23^{rd} day of February of the year of our Lord 2026

1 *Answer each of the following questions by marking either True or False but not both. You may assume all of the axioms of Zermelo-Fraenkel set theory.*

1. If this sentence is *false*, then $\varnothing$ is empty.

   ○ *True*
   ○ *False*

2. $\big\{\{\varnothing, \{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\big\}$ is a natural number.

   ○ *True*
   ○ *False*

3. $\mathbb{P}(\varnothing)$ is a subset of every set.

   ○ *True*
   ○ *False*

4. $\forall x \exists y (y \in x)$.

   ○ *True*
   ○ *False*

5. $\forall x \exists y (y \subseteq x)$.

   ○ *True*
   ○ *False*

6. This sentence has no truth value.

   ○ *True*
   ○ *False*

7. $\exists x \exists y \exists z \big((x \in y) \land (y \in z) \land (z \in x)\big)$.

   ○ *True*
   ○ *False*

8. $\exists x \exists y \exists z \big((x \subseteq y) \land (y \subseteq z) \land (z \subseteq x)\big)$.

   ○ *True*
   ○ *False*

9. $\Big\{x \;\Big|\; \exists y \big(y = \{z \mid \exists w (w \in x \land z \in w)\}\big)\Big\}$ exists.

   ○ *True*
   ○ *False*

10. The sentence given in problem 6 on this page has no truth value.

    ○ *True*
    ○ *False*

2 *The axioms, rules of inference, and theorems of the zeroth-order and first-order logic must be explicitly cited when used.*

For any propositions α, β, γ, prove the statement shown below.

$$\alpha \vee \beta,\ \beta \to \alpha,\ \alpha \to \gamma \ \vdash\ \gamma$$

3 *The axioms, rules of inference, and theorems of the zeroth-order and first-order logic **need not** be cited. You may assume axioms 0 through 6 of set theory, and you may rely on any theorems proven in lectures, notes, and problem sets.*

Show that $\forall x \forall y (x \cap y = \varnothing \implies x \subseteq x \setminus y)$.

4 *The axioms, rules of inference, and theorems of the zeroth-order and first-order logic* ***need not*** *be cited. You may assume axioms 0 through 6 of set theory, and you may rely on any theorems proven in lectures, notes, and problem sets.*

Show that $\forall x \forall y \big(\mathbb{P}(x) \subseteq \mathbb{P}(y) \Rightarrow x \subseteq y \big)$.