Esoterica 0

Discrete Structures

2nd day of February of the year of our Lord 2026

We will work up to a proof of $\forall x(\varphi(x)) \equiv \neg\exists x(\neg\varphi(x))$ and $\exists x(\varphi(x)) \equiv \neg\forall x(\neg\varphi(x))$.

1. Let φ be a unary predicate.
   a. Show that $\exists x(\neg\varphi(x)) \vdash \neg\forall x(\varphi(x))$.
   b. Show that $\forall x(\neg\varphi(x)) \vdash \neg\exists x(\varphi(x))$.
2. Let φ be a unary predicate.
   a. Show that $\neg\exists x(\varphi(x)) \vdash \forall x(\neg\varphi(x))$.
   b. Show that $\neg\forall x(\varphi(x)) \vdash \exists x(\neg\varphi(x))$.

Now we will show two important theorems. Pay attention to the parentheses.

3. Let φ be a unary predicate and let ψ be a proposition.
   a. Show that $\forall x(\varphi(x) \to \psi) \vdash \exists x(\varphi(x)) \to \psi$.
   b. Show that $\exists x(\varphi(x)) \to \psi \vdash \forall x(\varphi(x) \to \psi)$.