Practice Midterm 2

Discrete Structures

1[st] day of May of the year of our Lord 2026

1 *Answer each of the following questions by marking True or False but not both.*

1. If X and Y are finite and $f : X \to Y$ is a surjection, then f is injective.

   ○ *True*
   ○ *False*

2. $\forall x \forall y \forall z \Big(\big(x \setminus \{z\} = y \setminus \{z\}\big) \Rightarrow \big(x = y\big) \Big)$.

   ○ *True*
   ○ *False*

3. For any set X, the mapping $\big\{ (x, \{x\}) \bigm| x \in X \big\}$ is a function from X to $\mathbb{P}(X)$.

   ○ *True*
   ○ *False*

4. If X and Y are sets and $\lambda : X \to Y$, then $|\lambda| = |X|$.

   ○ *True*
   ○ *False*

5. $\forall x \exists y \big(|x| > |y| \big)$.

   ○ *True*
   ○ *False*

6. If A and B are sets such that $A \subseteq B$ and $A \neq \varnothing$, then $|B \setminus A| < |B|$.

   ○ *True*
   ○ *False*

7. It is possible to construct a graph G such that $\big(\forall v \in V(G)\big)\big(\mathtt{deg}_G(v) = 1\big)$.

   ○ *True*
   ○ *False*

8. It is possible to construct a graph G with n nodes and 2^n edges for every $n \in \mathbb{N}$.

   ○ *True*
   ○ *False*

9. The set of all finite binary strings is uncountably infinite.

   ○ *True*
   ○ *False*

10. If X and Y satisfy $|X| = |Y|$ and $f : X \to Y$ is an injection, then f is also a surjection.

    ○ *True*
    ○ *False*

2 *You may assume all axioms and theorems we have discussed so far, including any basic arithmetic and algebraic properties of* $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, *and* $\mathbb{R}$.

Consider the recursively defined function $f : \{2^x \mid x \in \mathbb{N}\} \to \mathbb{N}$ described below.

$$f(n) := \begin{cases} 1 & \text{if } n = 1 \\ 2 \cdot f\left(\frac{n}{2}\right) + n & \text{if } n > 1 \end{cases}$$

Prove that $f(n) = n \cdot \log_2(n) + n$ for all $n \in \{2^x \mid x \in \mathbb{N}\}$.

3 *You may assume all axioms and theorems we have discussed so far, including any basic arithmetic and algebraic properties of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.*

Consider the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ given by $f(x, y) := 3^x 5^y$ for $(x, y) \in \mathbb{N} \times \mathbb{N}$.
Prove that f is injective.

4 *You may assume all axioms and theorems we have discussed so far, including any basic arithmetic and algebraic properties of* $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, *and* $\mathbb{R}$.

Prove that there are an even number of odd-degree nodes in any graph.